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Workers trying to climb the income ladder: debt-financed knowledge capital accumulation and distributive conflict

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Abstract

This paper explores the two-way relationship between workers' investment in knowledge capital, funded through income-contingent loans, and functional income distribution within a demand-led macrodynamic framework. Real wage growth is positively influenced by the employment rate (which equals the utilization rate of knowledge capital) and the growth of labor productivity (which equals the growth of knowledge capital), and negatively by the ratio of workers' debt to knowledge capital. In the short-run equilibrium, the level of economic activity is positively influenced by the labor share of income. In the unique long-run equilibrium—which is stable as long as the response of real wage growth to the workers' debt ratio is not too strong—parameter changes that boost aggregate demand or strengthen workers' bargaining power in wage negotiations (with the exception of the parameter mediating the positive effect of the expected real wage income on investment in knowledge capital) result in a higher labor share and a lower workers' debt ratio. While greater investment in knowledge capital raises the level of economic activity in the short-run equilibrium, it may not have the same effect—and may fail to increase the labor share—in the long-run equilibrium.

Keywords

Knowledge capital, demand-led model, workers' indebtedness, wage bargaining.

JEL Code

E12; E22; E24.

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Resumo

Este artigo explora a relação bidirecional entre o investimento dos trabalhadores em capital conhecimento, financiado por meio de empréstimos com pagamento contingente à renda, e a distribuição funcional da renda dentro de um arcabouço macrodinâmico liderado pela demanda. O crescimento do salário real é positivamente influenciado pela taxa de emprego (que equivale à taxa de utilização do capital conhecimento) e pelo crescimento da produtividade do trabalho (que equivale ao crescimento do capital conhecimento), e negativamente influenciado pela razão entre a dívida dos trabalhadores e o capital conhecimento. No equilíbrio de curto prazo, o nível de atividade econômica é positivamente influenciado pela participação do trabalho na renda. No único equilíbrio de longo prazo—que é estável desde que a resposta do crescimento do salário real à razão de endividamento dos trabalhadores não seja muito forte—mudanças nos parâmetros que impulsionam a demanda agregada ou fortalecem o poder de barganha dos trabalhadores nas negociações salariais (com exceção do parâmetro mediador do efeito positivo da renda salarial real esperada sobre o investimento em capital conhecimento) resultam em uma maior participação do trabalho na renda e uma menor razão de endividamento dos trabalhadores. Embora um maior investimento em capital conhecimento eleve o nível de atividade econômica no equilíbrio de curto prazo, esse efeito pode não se manter—e pode não aumentar a participação do trabalho na renda—no equilíbrio de longo prazo.

Palavras-chave

Capital conhecimento, modelo liderado pela demanda, endividamento dos trabalhadores, barganha salarial.

1 Introduction

Student loans have contributed significantly to the growth of schooling, knowledge capital formation, and the higher education sector in several countries in recent decades. However, despite its contribution to a more skilled labor force, there is evidence that this funding mechanism has an important downside: the rise in household indebtedness, which has significantly impacted labor market participation through multiple interconnected channels. For instance, indebtedness may compel individuals to accept lower wages or less suitable jobs due to the urgency of repaying or rolling over their loans, thereby weakening their bargaining position in nominal wage negotiations.[1] This inevitable tension is particularly pertinent in the context of income-contingent repayment schemes (which set monthly payments based on a borrower's income), such as those implemented in the U.S., which, while mitigating default risk, still tether labor market decisions to debt obligations.[2]

[1] An essential tenet of Keynesian macroeconomics, derived directly from Keynes (1936), is the idea that the wage bargain is conducted in nominal terms, with the real wage being determined only after the wage bargain is complete, depending on how the price level adjusts.

[2] In the U.S., federal student loans can be forgiven, canceled, or discharged under certain conditions, relieving borrowers of some or all of their repayment obligations. Serra (2023) explores the macroeconomic effect of this policy using a demand-led dynamic model. However, the study does not consider how a reduced student debt burden might influence workers' bargaining power in wage negotiations.

Building upon this context, this paper examines the distributive dimensions of knowledge capital accumulation financed through student loans. On the one hand, this paper considers the positive effect of knowledge capital accumulation on labor productivity and workers' bargaining power in nominal wage negotiations, as explored in Lima et al. (2021). On the other hand, this paper also examines how household debt negatively affects workers' bargaining power over wages. In fact, the urgency to secure and maintain employment (often leading workers to accept job positions that do not align with their educational qualifications) can be driven by the impending start of loan repayment obligations (Ji, 2021). This situation is particularly evident in the context of student loans, which are typically non-dischargeable in bankruptcy and thus impose persistent financial pressure on borrowers. Regarding household debt more generally, there is an increased aversion to the risk of losing labor income, given the obligation to service the outstanding debt (Kim et al., 2019), and empirical evidence shows that higher levels of household indebtedness are associated with lower unionization rates (Gouzoulis, 2024). Therefore, working households' debt not only affects individual financial stability but also has broader implications for labor market dynamics and workers' collective bargaining power.

Mainstream economic growth models, following the pioneering traditions of Lucas (1988), Romer (1990), and Mankiw et al. (1992), focus exclusively on the supply-side aspects of labor-augmenting investment in human capital, abstracting from its demand-side effects. However, as a costly investment with relevant implications for the functional distribution of income – and thus consumption and investment in physical capital accumulation – knowledge capital investment contributes to aggregate demand formation and hence crucially affects the dynamics of the level of economic activity in a closed economy. In addition, the mainstream growth literature largely neglects the complex and multifaceted roles played by knowledge capital and debt accumulation in (nominal) wage negotiations, especially their potential to both enhance and weaken the bargaining power of workers (Lima et al., 2024).

In the non-mainstream literature, Dutt (2010) presents a demand-led model to assess the economic consequences of skill acquisition and wage dynamics. As in Dutt (2010), the model set forth herein formally explores theoretical underpinnings and implications of knowledge capital accumulation within a demand-led dynamic model, but focusing on a rather different set of transmission channels and mechanisms. Considering that the making of costly investments in knowledge capital formation through education is one of the main varieties of productivity-enhancing knowledge capital accumulation, this paper explores several macroeconomic implications of the accumulation of knowledge capital by workers as mediated by its expansionary effect on labor productivity in a demand-led dynamic model of (physical and knowledge) capital capacity utilization and output growth.

This paper also examines the closely related issue of how debt-financed investment in knowledge capital affects and is affected by the functional distribution of income between wages and profits within a macrodynamic model driven by aggregate effective demand. It considerably extends three analytical contributions from the existing related literature: Lima et al. (2021), which analyzes a conflicting-claims distributive dynamic (a bargaining process in nominal wage negotiations) within a public universal education system; and Serra (2023) and Carvalho et al. (2024), which explore the dynamics of human or knowledge capital and household indebtedness in a model setting where education is funded through student loans. The motivation here lies in the income distributive implications of this semi-self-financing method of investing in knowledge capital formation, particularly its impact on workers' bargaining power in nominal wage negotiations. A central takeaway is that, as with physical capital and the labor force, the mere availability of knowledge capital does not automatically generate corresponding demand, which has important implications for the dynamics of the labor share of income.

Accordingly, several factors influencing the growth rate of nominal wages will be taken into account. First, the positive effect of labor market tightness (as measured by the employment rate) on workers' bargaining power, as widely discussed in the literature (see, e.g., Lima et al., 2021; Sasaki, 2013; Tavani et al., 2011; Cassetti, 2002; Skott, 1989; Blecker and Setterfield, 2019; Setterfield, 2023). Second, the positive contribution of knowledge capital accumulation to the bargaining power of workers with relatively higher qualification levels, including their greater awareness of the corresponding productivity gains (Lima et al., 2021). Third, as originally formally examined by Kim et al. (2019), household indebtedness negatively affects workers' wage bargaining power, due to the perceived risk of being unable to service outstanding debt (or to roll it over on reasonable terms) in the event of unemployment. Therefore, by increasing the perceived cost of job loss, household indebtedness functions as an additional mechanism through which unemployment disciplines workers in nominal wage negotiations.

We model the macrodynamics of a closed economy in which the government does not directly participate in the income-expenditure circuit or output production, and where debt-financed accumulation of knowledge capital by working households contributes to labor productivity growth. Our aim is not to replicate or explain the U.S. experience with student loans. Instead, the model draws some indirect motivation from that context by incorporating an Income-Driven Repayment Plan, which currently represents the most common student debt repayment scheme in the U.S. Real wage growth is determined through a conflicting-claims bargaining process, in which workers' bargaining power depends on the rate of knowledge capital accumulation (which determines labor productivity growth), as well as the employment rate and a debt ratio measuring working households' indebtedness.

In the short-run equilibrium configuration, wage-led economic activity occurs alongside multiplier and accelerator effects driven by both consumption and investment demand. In the long run, rising working households' indebtedness may erode workers' bargaining power, potentially leading to real wage restraint and causing macroeconomic instability. In the long-run equilibrium, parameter changes that stimulate aggregate demand improve labor market conditions, enhance workers' bargaining power, and raise the labor share of income. In contrast, a stronger knowledge capital accelerator effect reduces the labor share by speeding up labor productivity growth. In general, with the exception of the parameter related to the physical capital accelerator effect, parameter changes that influence the labor share also affect the knowledge-to-physical capital ratio—an indicator of the composition of the economy's total capital stock or wealth—in the same direction. Parameter changes that stimulate aggregate demand or strengthen workers' bargaining power in nominal wage negotiations potentially allow workers to finance knowledge capital formation with reduced reliance on debt. As a result, positive shocks to these parameters (with the exception of the parameter mediating the positive effect of the expected real wage income on investment in knowledge capital) increase the labor share and lower the debt ratio of working households—measured relative to the stock of knowledge capital, and thus interpretable as their leverage ratio.

The remainder of this paper is structured as follows. Section 2 introduces a macrodynamic model driven by aggregate effective demand that examines the interplay between knowledge capital accumulation, debt-financed investment in knowledge capital formation, and a conflicting-claims bargaining process determining the real wage and labor share of income. Sections 3 and 4 present the formal results of the model, addressing both the short-run and long-run dynamics, respectively. Section 5 provides an analytical discussion of the primary findings from the formal analysis and Section 6 concludes the paper, summarizing the key insights and implications.

2 Structure of the model

The economy represented in this model is closed and produces a single good/service that is used flexibly for consumption and investment. Workers invest in the formation of their own knowledge capital by financing it through debt, thereby employing a semi-self-financing strategy, as in Carvalho et al. (2024). The desired investment of workers in knowledge capital is specified as an independent investment function, it being accommodated without any quantity rationing by an endogenous supply of credit, as in Carvalho et al. (2024) and Serra (2023). Working households buy a flow of the single good/service from firms and use it either for consumption or investment in knowledge capital formation. In the long-run equilibrium, the growth rate of final output is determined by the growth rate of both physical and knowledge capital, as both will grow at the same rate.

Firms employ physical capital and labor (where the latter is endowed with knowledge capital) to produce the single good/service. By virtue of being non-excludable and nonrival, knowledge capital is always uniformly distributed across the labor force. The production process uses a fixed-coefficient technology to combine these inputs:

$$X = \min\{Kv, La(H)\}, \tag{1}$$

where X is the level of production, K is the physical capital stock, L is the level of employment, v is an exogenous parameter representing the ratio between the output level at full capacity and the physical capital stock, H is the stock of knowledge capital, and $a(H) = X/L$ is the ratio of output to labor (i.e., average labor productivity), which is a function of the knowledge capital stock, with $a(0) = 0$, $a'(H) > 0$, and $a''(H) \leq 0$. Two convenient (and mostly innocuous) normalizations are made. First, for the technical coefficient, $v = 1$, so that $u = X/K$ measures the rate of capacity utilization. Second, the size of the labor force is also normalized to one ($N = 1$), which results in H also representing the average stock of knowledge capital. Given that H is uniformly distributed across the labor force, unemployed workers are as knowledge capital endowed as employed ones, so that $e = L/N$ represents both the employment rate and the utilization rate of the knowledge capital stock. Also, the stocks of physical and knowledge capital do not depreciate, and unemployment does not cause labor deskilling.

The economy is structured around two primary social classes, capitalists and workers. Capitalists own the firms and receive profits, whereas workers supply labor and earn wages. Aggregate income is entirely distributed between these two classes:

$$X = VL + R, \tag{2}$$

where V is the real wage, which is given (or predetermined) in the short run, and R is the aggregate level of real profits. From (1) and (2), the labor share of income, σ, is:

$$\sigma = \frac{V}{a(H)}. \tag{3}$$

Following the Principle of Effective Demand, firms produce according to the level of aggregate effective demand. Considering that firms operate below full capacity in both physical capital and labor (where the latter includes the knowledge capital embodied within it), it follows from (1) that the employment level is given by:

$$L = \frac{X}{a(H)}. \tag{4}$$

Using (1) and (4), the employment rate is given by:

$$e = \frac{L}{X}\frac{X}{K}\frac{K}{N} = \frac{u}{h}, \tag{5}$$

where $h = Na(H)/K$ represents the ratio of labor supply in effective (or productivity) units to the physical capital stock. As the average labor productivity is given by $a(H) = H$ and $N = 1$, it follows that $h = H/K$, which is given (or predetermined) in the short run.

Firms make decisions about accumulating physical capital independently of prior savings. As in Carvalho et al. (2024), the desired growth rate of the physical capital stock, g_K, assuming no depreciation, is expressed as:

$$g_K = \frac{I_K}{K} = \beta u, \tag{6}$$

where β is a strictly positive parameter and I_K represents firms' desired level of investment in physical capital.

In the short run, the technological parameter $a(H)$ is given, resulting from previous dynamics of accumulation of knowledge capital. Considering that the average labor productivity is given by $a(H) = H$, it grows at the same rate as the average knowledge capital, H:

$$\hat{a} = \hat{H} = g_H, \tag{7}$$

where g_H is the growth rate of the average knowledge capital.

Note that, given (3), which can be re-written as $\sigma = V/H$, the labor share of income serves as a measure of the wage compensation workers receive for their knowledge capital. A constant labor share indicates that the conflict between workers and capitalists over income shares results in an increase in labor productivity from knowledge capital accumulation being fully passed on to real wages.

Similar to the determination of investment in physical capital, which is independent of prior savings, the decisions of working households to accumulate knowledge capital are also assumed to be independent of their own funding. This autonomy in accumulating knowledge capital is enabled by an endogenous supply of credit money that flexibly adapts to demand, rendering the investment in this additional capital asset predominantly debt-financed. Working households' desired level of investment in knowledge capital is given by:

$$I_H = \gamma V L, \tag{8}$$

where $I_H = \dot{H} = dH/dt$ is the rate of change in the knowledge capital stock in the absence of depreciation, VL, the real wage bill, is a proxy for the expected conditions of labor remuneration and employment in the relevant future, and γ is a strictly positive parameter. As in the Cambridge U.K. tradition, where current or expected total profits typically determine the desired investment in physical capital, workers' desired level of investment in knowledge capital, as specified in (8), is positively related to the wage bill as a proxy for the expected labor market conditions. Using (3) and (5), the rate of growth of the knowledge capital stock, and hence in the average knowledge capital, g_H, is given by:

$$g_H = \frac{\dot{H}}{H} = \frac{I_H}{H} = \gamma \frac{VL}{H} = \gamma\sigma e = \gamma\sigma\frac{u}{h}. \tag{9}$$

Note that, akin to the desired investment in physical capital in (6), the specification in (9) features an accelerator effect, albeit in terms of the utilization rate of the knowledge capital stock (or the employment rate, e). This accelerator effect intuitively increases with the labor share. Note also that household indebtedness does not explicitly enter the desired investment in knowledge capital, as shown in (8) and (9). This modeling specification is partially motivated by the U.S. experience with federal student loans, where outstanding debt balances do not, in isolation, prevent individuals from securing new loans. However, as discussed later, indebtedness indirectly dampens investment in knowledge capital formation in the long run by negatively affecting labor income.

For reference and use shortly, the ratio of investment in knowledge capital to physical capital is given by:

$$\frac{I_H}{K} = \gamma \frac{VL}{X}\frac{X}{K} = \gamma\sigma u. \tag{10}$$

3 Short-run equilibrium

In the short run, the stock of physical capital, K, the stock of knowledge capital, H, which is equal to the average knowledge capital across the labor force (recall that the size of the labor force, N, is normalized to one), and working households' debt, D, as well as the real wage, V, and the average labor productivity, a, which is equal to the average knowledge capital, H, are all given (or predetermined). In the market clearing equilibrium, aggregate supply has become equalized to aggregate demand through adjustments in the rate of capacity utilization, u. The equilibrium in the product market is given by:

$$X = C_w + C_p + I_H + I_K, \tag{11}$$

where C_w and C_p are the levels of consumption of workers and capitalists, respectively. Note that I_H constitutes an additional source of aggregate demand, complementing consumption and investment in physical capital.

Workers borrow to finance their desired level of investment in knowledge capital. The resulting debt servicing follows an income-driven repayment plan, akin to the U.S. federal Income-Based Repayment (IBR) plan for student loans. This plan stipulates that debtors allocate a fixed percentage of their disposable income flow (here, represented by $0 < 1-\phi < 1$) toward servicing the debt. Therefore, workers consume the remaining fraction ($0 < \phi < 1$) of their disposable income flow after debt servicing, as represented by:

$$C_w = VL - (1-\phi)VL = \phi VL, \tag{12}$$

which can be expressed as a ratio relative to the physical capital stock as follows:

$$\frac{C_w}{K} = \phi \frac{VL}{X}\frac{X}{K} = \phi u \sigma. \tag{13}$$

Note that, since saving represents abstention from consumption, the fraction $1-\phi$ denotes workers' propensity to save from their disposable income, which is allocated toward debt servicing. As a result, the rate of change in the stock of debt held by workers equals interest payments plus new investment in knowledge capital, minus the portion of workers' real wage income allocated toward debt servicing. Using (8) and (12), the rate of change in the stock of debt held by workers is then given by:

$$\dot{D} = iD - (1-\phi)VL + I_H = iD + [\gamma - (1-\phi)]VL, \tag{14}$$

where $\dot{D} = dD/dt$ is the rate of change in the stock of debt held by workers, and $i > 0$ is an exogenously given interest rate. As workers allocate a constant fraction of their disposable income to debt servicing, (14) indicates that debt amortization occurs only if $(1-\phi)VL$ exceeds the sum of interest payments, iD, and new investment in knowledge capital, I_H. Combining (8) and (14), it is evident that workers can pay at least part of the interest service on their outstanding debt only if the responsiveness of their investment in knowledge capital, given by γ in (8), is strictly lower than the share of income allocated to debt repayment, given by $1-\phi$ in (12). Notably, under the IBR framework, the debt repayment share remains constant regardless of the debt service on outstanding balance. As properly discussed later, the parameter configuration of the model ensures that working households remain indebted in the long-run equilibrium. While it is possible that $(1-\phi)-\gamma > 0$, it is insufficiently large to result in $\dot{D} < 0$, though.

Using (14), the growth rate of the stock of debt is given by:

$$\hat{D} = \frac{\dot{D}}{D} = i - (1-\phi)\frac{VL}{X}\frac{X}{K}\frac{K}{H}\frac{H}{D} + \frac{I_H}{H}\frac{H}{D} = i - (1-\phi)\frac{\sigma u}{h\delta} + \gamma\frac{\sigma u}{h\delta} = i - (1-\phi-\gamma)\frac{\sigma u}{h\delta}, \tag{15}$$

where $\delta = D/H$ is the ratio of working households' debt stock to their knowledge capital stock. This ratio is intuitive, as the stock of knowledge capital can be interpreted as constituting the wealth of working households.

Capitalists derive income from both profits (through ownership of firms) and interest payments on loans extended to workers (Dutt, 2006; Kapeller and Schütz, 2015). The latter source of income arises from capitalists' participation as financial rentiers in a credit system, which we innocuously abstract from modeling. Capitalists save a constant fraction $0 < s < 1$ of their disposable income, allocating the remaining part to consumption:

$$C_p = (1-s)[(1-\sigma)X + iD], \tag{16}$$

which can be represented as a ratio relative to the physical capital stock as follows:

$$\frac{C_p}{K} = (1-s)[(1-\sigma)u + i\delta h]. \tag{17}$$

As in Kaldor (1955), Kalecki (1971), Robinson (1962), and Pasinetti (1962), workers and capitalists have different consumption behaviors, so that $s > 1-\phi$, considering that the fraction $1-\phi$ in (12) can be interpreted as workers' propensity to save (and hence not to consume) out of disposable income. Notice that the part of the wage bill that is not devoted to consumption represents an income transfers from working households to capitalists. Nevertheless, the total interest income accruing to capitalists is given by iD, the entire debt service in the absence of debt amortization, as specified in (16). Any excess of the combined debt service and investment in knowledge capital over the portion of the wage bill allocated to debt repayment leads to an increase in the stock of debt held by working households, as specified in (14).

Dividing (11) by K, and substituting (6), (10), (13), and (17) into (11), we can then compute the rate of capacity utilization in the short-run equilibrium as:

$$u^* = \frac{(1-s)i\delta h}{s(1-\sigma) + (1-\phi-\gamma)\sigma - \beta} = \frac{(1-s)i\delta h}{\Omega}, \tag{18}$$

where the parameter configuration, including $s > \beta$, and the economically relevant values for the labor share of income, $0 < \sigma < 1$, together ensure that $\Omega \equiv s - \beta - (s+\phi+\gamma-1)\sigma > 0$, which guarantees that the solution in (18) is strictly positive and stable. This Keynesian stability condition requires that, all else constant, aggregate demand leakages be more responsive to changes in the rate of capacity utilization than aggregate demand injections, so that any excess aggregate demand or supply be eliminated instead of exacerbated by changes in the rate of capacity utilization. Furthermore, given that working households allocate a greater proportion of their income to consumption compared to capitalist households, $\phi > 1-s$, it follows that $s+\phi+\gamma-1 > 0$. This, in turn, implies that the short-run equilibrium capacity utilization rate in (18) is positively affected by the labor share.

In Appendix A.1, we perform the corresponding comparative statics exercises. Consistent with a demand-led model, the short-run equilibrium value of the rate of capacity utilization in (18) is positively influenced by the parameters β and γ, which represent the accelerator effects in the functions for the desired rate of growth of physical and knowledge capital in (6) and (9), respectively. Conversely, this endogenous measure of the level of economic activity is negatively influenced by the saving rate of capitalists, s, and the workers' repayment coefficient, $1-\phi$, in the short-run equilibrium.

The positive influence exerted by the interest rate, i, and the debt ratio, δ, are attributed to their positive contribution to capitalists' consumption as a proportion of the physical capital stock (per (17)), and their lack of influence on workers' consumption as a proportion of the physical capital stock (per (13)), owing to the operation of the IBR plan. Recall that under this plan, workers' repayments are structured to insulate their consumption from exogenous changes in the interest rate and debt ratio. Additionally, the ratio of knowledge to physical capital, h, has a positive influence on the short-run equilibrium value of the rate of capacity utilization through its influence on capitalists' consumption as a proportion of the physical capital stock (per (17)), and its lack of influence on workers' consumption as a proportion of the physical capital stock (per (13)).[3]

Meanwhile, an increase in the labor share redistributes income from capitalist households, who allocate a strictly positive proportion of their total income (including profit and interest) to savings, directly to working households, who in turn spend all their net wage income on consumption. Besides, a higher labor share intensifies the accelerator effect associated with the desired rate of knowledge capital accumulation in (9). This redistribution increases aggregate demand, thereby raising the rate of capacity utilization in the short-run equilibrium.

Aside from the influence of the knowledge-to-physical capital ratio, h, all the signs for the comparative statics results discussed above for the short-run equilibrium capacity utilization rate also apply to the short-run equilibrium employment rate. This equivalence can be verified by substituting (18) into (5), which allows noting that the short-run equilibrium employment rate given by $e^* = u^*/h$ does not depend on h. Appendix A.1 also presents a summary of the signs for all comparative statics exercises concerning the short-run equilibrium values of both the rates of capacity utilization and employment in Table 1.

4 Long-run equilibrium

In the long run dynamics, the short-run equilibrium values of capacity utilization and employment, as determined in the preceding section, are consistently attained. These equilibrium values are parameterized by the debt-to-knowledge capital ratio, δ, the knowledge-to-physical capital ratio, h, and the labor share of income, σ. These predetermined variables evolve toward their long-run levels in response to changes in the stocks of physical capital, K, knowledge capital, H, and debt, D, for the first two ratios, and in response to changes in real wage and labor productivity for the labor share. Per (7), changes in the stock of knowledge capital also influence average labor productivity, a, while the dynamics of the real wage are driven by conflicting claims over available income. Hence, the long-run analysis entails a three-dimensional dynamic system involving h, δ, and σ. This framework integrates a conflicting-claims determination of the functional distribution of income, extending the simpler model structure proposed by Lima et al. (2021), with debt-financed knowledge capital accumulation by workers, as developed by Carvalho et al. (2024), along with several additional components.

At any given point in time (referred to as the short run), the real wage is predetermined. Since labor is always in excess supply, employment is determined solely by labor demand. Over time, the real wage evolves at a rate aligned with workers' desired growth rate, $\hat{V}$, within an accelerationist framework characterized by conflicting claims on available real income.[4] The

[3] The influence of the debt ratio, δ, and the knowledge to physical capital ratio, h, on the short-run equilibrium value of the rate of capacity utilization in (18) should be interpreted with caution to prevent potential misunderstandings. The influence of $\delta = D/H$, all else constant (including $h = H/K$), should ultimately be interpreted as the influence of the stock of working households' debt, D. Meanwhile, the influence of h, all else constant (including δ), should ultimately be interpreted as the influence of the stock of physical capital, K.

[4] Note that specifying wage dynamics in terms of the real wage is not logically inconsistent with the point emphasized in footnote 1—that the wage bargain is conducted in nominal terms, with the real wage subsequently determined by how the price level adjusts.

real wage dynamics is described by:

$$\hat{V} = \lambda_1 e + \lambda_2 \hat{a} - \lambda_3 \delta, \tag{19}$$

where $\lambda_1 > 0$ is a parameter denoting the sensitivity of workers' desired real wage growth to the state of the labor market, as measured by the employment rate, e. A higher employment rate increases real wage growth by improving workers' bargaining power, as specified in Lima et al. (2021) and Blecker and Setterfield (2019). Workers' desired real wage growth, which translates into actual real wage growth, also depends positively on the rate of growth of labor productivity, $\hat{a}$. This, in turn, is a positive function of the rate of growth of average knowledge capital, $\hat{H}$ (recall that the size of the labor force is normalized to one), so that $\lambda_2 > 0$, as specified in Lima et al. (2021). Finally, now inspired by Kim et al. (2019), but positing the substance of the argument with a different formal specification, real wage growth depends negatively on the ratio of debt-to-knowledge capital, δ.[5] Household debt and workers' borrowing behavior impact the labor market by influencing the cost of job loss, which, in turn, affects workers' bargaining power in nominal wage negotiations. Higher indebtedness (i.e., a higher debt-to-knowledge capital ratio) increases the cost of losing a job, as workers need to service their debt obligations. This heightened cost reduces workers' bargaining power in wage negotiations, negatively affecting real wage growth, so that $\lambda_3 > 0$. Importantly, this reduced-form bargaining power effect owing to higher indebtedness is not entirely correlated with the employment rate.

The behavior of the economy toward the long run can be investigated by focusing on the dynamics of the state variables $h = H/K$, $\delta = D/H$ and $\sigma = V/a$. Using (5), (6), (9),(15), and (19), the dynamics of these state variables are represented by the following three-dimensional system of differential equations:

$$\hat{h} = \hat{H} - \hat{K} = \gamma\sigma\frac{u}{h} - \beta u, \tag{20}$$

$$\hat{\delta} = \hat{D} - g_H = \left[i - (1 - \phi - \gamma)\sigma\frac{u}{h\delta}\right] - \gamma\sigma\frac{u}{h}, \tag{21}$$

$$\hat{\sigma} = \hat{V} - \hat{a} = \left[\lambda_1\frac{u}{h} + \lambda_2\frac{\gamma\sigma u}{h} - \lambda_3\delta\right] - \left[\frac{\gamma\sigma u}{h}\right], \tag{22}$$

where u is given by (18).

In Appendix A.2.1, we use the Routh-Hurwitz conditions to analyze the stability properties of the unique long-run equilibrium configuration of the three-dimensional dynamic system in (20)-(22). This long-run equilibrium solution characterized by $\hat{h} = \hat{\delta} = \hat{\sigma} = 0$ is locally stable, the reason being that the parameter configuration ensures a value of λ_3 low enough in (19) such that $\partial\hat{\sigma}/\partial\sigma < 0$. Therefore, the sufficiently low intensity of the response of the real growth of

[5] In the model set forth in Kim et al. (2019), which does not consider knowledge capital accumulation, the rate of change in the labor share of income, $\dot{\sigma} = d\sigma/dt$ (using our notation), depends negatively on the ratio of working households' debt to physical capital. Since our model features two accumulable capital assets (physical and knowledge) and therefore the operation of additional transmission channels, whereas the knowledge capital stock, H, can be intuitively interpreted as constituting the wealth of working households, we adopt the following decomposition: $D/K = (H/K)(D/H) = h\delta$. While h is a measure of the composition of the total capital stock or wealth of the economy, δ can be interpreted as the leverage ratio of working households.

the real wage to a change in the ratio of working households' debt to their knowledge capital ensures that the unique long-run equilibrium solution of the three-dimensional dynamic system in (20)-(22) is locally stable.

Consistent with the findings of Kim et al. (2019), our analysis highlights the pivotal role of working households' debt as a potential source of macroeconomic instability. However, as noted in footnote 3, while Kim et al. (2019) define the corresponding debt ratio by normalizing working households' debt by the physical capital stock, our model instead normalizes it by the knowledge capital stock. This alternative specification yields a ratio that is more readily and intuitively interpreted as a leverage rate (but recall that the model set forth in Kim et al. (2019) does not consider knowledge capital accumulation).

The qualitative finding above that household indebtedness can destabilize economic dynamics aligns with similar results in the existing literature. For example, Kim et al. (2019) examine a standard debt repayment framework and argue that when workers strive to maintain a target consumption level, the combination of rising debt ratios and declining wages can trigger additional borrowing, ultimately threatening debt sustainability. Similarly, Setterfield et al. (2016) incorporate household borrowing to support consumption patterns that emulate those of rentiers. They show that increased debt servicing shifts income from working households to rentiers, which can reduce aggregate consumption and lead to destabilizing effects, depending on how debt is serviced. In addition to the previously discussed differences in knowledge capital accumulation, our model incorporates repayment specified by an IBR plan, which implies that higher debt balances do not automatically translate into higher debt servicing obligations for working households. While this feature may help mitigate the destabilizing effects identified by Kim et al. (2019) and Setterfield et al. (2016), our findings also point to a persistent vulnerability to instability associated with household indebtedness. This highlights the significance of the channel through which rising debt erodes workers' bargaining power in wage negotiations, which is mediated by $\lambda_3 > 0$ in (19).

If the responsiveness of the rate of growth of the real wage to the ratio of working households' debt to their knowledge capital (captured by λ_3 in (19)), ceases to be sufficiently weak, such that $\partial\hat{\sigma}/\partial\sigma > 0$, macroeconomic instability emerges. A higher debt ratio, for example, by weakening workers' bargaining power in nominal wage negotiations, reduces their ability to service outstanding debt and encourages further borrowing to meet debt obligations. In addition, the fall in the growth rate of the real wage adversely affects both economic activity and human capital accumulation, thereby exerting further upward pressure on the debt ratio.

As shown in Appendix A.2.1, the condition that ensures $\partial\hat{\sigma}/\partial\sigma < 0$ is $\lambda_3 < \lambda_1(1-s)i/(s-\beta)$. Therefore, the higher the saving rate of capitalist households, s, and the lower the interest rate, i, the responsiveness of the real wage growth to the employment rate, λ_1, and the strength of the accelerator effect in physical capital accumulation, β, the weaker the required responsiveness of the real wage growth rate to the ratio of worker's debt to knowledge capital, λ_3, must be in order to prevent macroeconomic instability.

Solving for $\hat{\sigma} = 0$ after substituting (18) into (22) yields the long-run equilibrium value of the labor share:

$$\sigma^* = \frac{\lambda_1(1-s)i - \lambda_3(s-\beta)}{(1-s)i\gamma(1-\lambda_2) - \lambda_3(s+\phi+\gamma-1)}. \tag{23}$$

As noted above, the parameter configuration of the model is such that the unique long-run equilibrium of the endogenous variables h, δ, and σ is locally stable, which ensures that the numerator of the expression in (23) is strictly positive. The parameter configuration also ensures that the denominator in the expression above remains strictly positive, as discussed in Appendix A.2.2, and that the resulting long-run equilibrium labor share takes only economically relevant values within the open interval $(0, 1)$.

Setting $\hat{h} = 0$ in (20), we get:

$$h^* = \frac{\gamma\sigma^*}{\beta}. \tag{24}$$

Given that the knowledge-to-physical capital ratio serves as an indicator of the composition of the economy's total capital stock, it is intuitive that, in the long-run equilibrium, this ratio exhibits a positive relationship with the labor share. However, the long-run equilibrium labor share is influenced by the parameters γ and β, as shown in (23). Therefore, the indirect effects of these parameters on the long-run equilibrium knowledge-to-physical capital ratio in (24) are mediated through their impact on the long-run equilibrium labor share, as discussed shortly.

Lastly, solving for $\hat{\delta} = 0$ after substituting (18) into (21) yields the long-run equilibrium value of the working households' debt ratio:

$$\delta^* = \frac{s[1 - (\phi + \gamma)\sigma^*] - \beta}{\gamma\sigma^*(1 - s)}. \tag{25}$$

It is intuitive that, in the long-run equilibrium, the debt ratio exhibits a negative relationship with the labor share. Nevertheless, the long-run equilibrium labor share is influenced by several parameters that also influence the long-run equilibrium debt ratio, as shown in (23) and (25). It follows that the indirect effects of these parameters on the long-run equilibrium debt ratio are mediated through their impact on the long-run equilibrium labor share, as discussed shortly.

5 Discussion

As previously discussed, the parameter $\lambda_3 > 0$, which mediates the negative effect of the debt ratio on the real wage growth rate in (19), may constitute a potential source of macroeconomic instability, depending on its magnitude. Moreover, this negative relationship introduces ambiguity in the effect of parameter changes on the long-run equilibrium values in (23)-(25). This ambiguity will become evident when we later set $\lambda_3 = 0$ for comparative purposes.

The effects of parameter variations on the long-run equilibrium labor share, with $\lambda_3 > 0$ but sufficiently small to ensure stability of the long-run equilibrium, are presented in Appendix A.2.2 and summarized in Table 2 in Appendix A.2.5. Among the bargaining power parameters that mediate the effect of changes in the determinants of real wage growth, a greater responsiveness of real wage growth to the employment rate, given by λ_1, and the growth rate of knowledge capital, measured by λ_2, raises the long-run equilibrium labor share, while a greater responsiveness to the debt ratio, given by λ_3, lowers it. The strength of the accelerator effect in the desired rate of physical (knowledge) capital accumulation, captured by the parameter β

(γ), is positively (negatively) associated with the long-run equilibrium labor share. Meanwhile, a higher workers' propensity to consume, ϕ, or a higher interest rate, i, positively influences the long-run equilibrium labor share, whereas an increase in the capitalists' saving rate has a negative effect.[6]

Regarding the knowledge-to-physical capital ratio in the long-run equilibrium, h^*, it is shown in Appendix A.2.3 that it varies negatively with the parameter measuring the strength of the accelerator effect in the desired rate of knowledge capital accumulation in (9), represented by γ. While this parameter change has a direct positive effect on h^*, it exerts an indirect negative effect by reducing the long-run equilibrium labor share in (23), as demonstrated in Appendix A.2.2. Overall, the indirect negative effect outweighs the direct positive one. The parameter β, which captures the strength of the accelerator effect in the desired rate of physical capital accumulation in (6), has an ambiguous effect on h^*. While an increase in β positively influences the long-run equilibrium labor share, as demonstrated in Appendix A.2.2, and hence exerts an indirect positive effect on h^*, it simultaneously has a direct negative effect on h^* in (24). For the remaining parameters that affect the long-run equilibrium labor share in (23), their indirect effect on the long-run equilibrium knowledge-to-physical capital ratio has the same sign as their direct effect on the labor share.

Recall that the debt ratio of working households, represented by $\delta = D/H$, can be interpreted as their leverage ratio. In fact, while workers cannot sell their knowledge capital directly to other workers, only to firms, they effectively leverage it by borrowing against expected future income derived from selling their labor services to firms, rendering δ a significant indicator of working households' financial exposure. In the long-run equilibrium solution, the debt ratio in (25) exhibits a negative relationship with the labor share. However, the long-run equilibrium labor share is influenced by several parameters that also influence the long-run equilibrium debt ratio, as shown in (23) and (25). As demonstrated in Appendix A.2.4, a higher propensity to consume among workers, ϕ, while indicating that they allocate a smaller proportion of their wage income to debt repayment, raises the labor share. This, in turn, boosts aggregate demand and capacity utilization, ultimately reducing the debt ratio. This result was first identified by Carvalho et al. (2024) and dubbed the *paradox of debt repayment*, a variant of the paradox of debt originally identified by Steindl (1952). The paradox herein arises because debt repayments constitute an income transfer from households that consume all of their income net of these repayments (workers) to those that save a portion of the repayments they receive (capitalists), thereby dampening aggregate demand.[7]

An increase in the parameter that captures the strength of the accelerator effect in physical capital accumulation, β, or knowledge capital accumulation, γ, has opposite effects on the long-run equilibrium debt ratio of working households: a higher β exerts a negative influence,

[6]This seemingly paradoxical result concerning the interest rate stems, in part, from the specific features of the repayment plan embedded in the model—a plan that requires workers to allocate a fixed percentage of their wage income to debt servicing. In the case of a standard repayment plan, Serra (2023) finds that a higher interest rate lowers the level of economic activity, potentially causing destabilizing effects. Building on Kim et al. (2019) to feature an extended specification of real wage growth, the model here introduces another potential source of instability: the direct effect of working household's debt on labor market dynamics.

[7]The paradox of debt repayment may not emerge in situations where working households have a strictly positive propensity to save. For example, if debt servicing reduces workers' savings while leaving their consumption unchanged, it may result in an aggregate demand injection—particularly if capitalists spend in consumption a portion of the debt payments they receive. Carvalho et al. (2024) examine different scenarios in which the paradox does not hold, depending on the pecking order by which consumption, debt servicing, and saving draw on wage income. The broader implications of how debt servicing behavior influences aggregate demand formation are thoroughly explored in Setterfield et al. (2016).

lowering the debt ratio, while a higher γ exerts a positive influence, raising it. In the long-run equilibrium, finally, a higher propensity to save by capitalists increases the debt ratio of working households by exerting downward pressure on aggregate demand formation and the labor share.

To gain further insight into the model's behavior through comparison, let us now explore the case where $\lambda_3 = 0$ in (19).[8] In this case, the bargaining power of workers (and consequently, real wage dynamics) is no longer affected by their debt ratio. As shown in Appendix A.3 (and summarized in Table 3), the long-run equilibrium labor share is no longer influenced by the parameters s, i, ϕ, and β. As before, both the knowledge-to-physical capital ratio and the debt ratio exhibit a positive relationship with the labor share in the long-run equilibrium. The latter continues to increase with the parameters λ_1 and λ_2, which reflect workers' bargaining power in the real wage dynamics as specified in (19), and to decrease with the parameter capturing the strength of the accelerator effect in the desired rate of knowledge capital accumulation, γ, as specified in (9). In relation to the long-run equilibrium knowledge-to-physical capital ratio, the influence of the parameter β, which captures the strength of the accelerator effect in the desired rate of physical capital accumulation, is no longer ambiguous and becomes negative. This is because changes in β no longer positively influence the long-run equilibrium labor share, which, when $\lambda_3 > 0$, helped to partially offset the direct negative effect of β on the long-run equilibrium knowledge-to-physical capital ratio. Moreover, the influence of the parameter γ, which captures the strength of the accelerator effect in the desired rate of knowledge capital accumulation, is no longer negative but becomes null. This is because, when $\lambda_3 = 0$, the long-run equilibrium knowledge-to-physical capital ratio no longer depends on γ. For the remaining parameters that affect the long-run equilibrium labor share when $\lambda_3 = 0$, which are λ_1 and λ_2, their indirect effect on the long-run equilibrium knowledge-to-physical capital ratio has the same positive sign as their direct effect on the labor share of income. Meanwhile, with respect to the debt ratio of working households in the long-run equilibrium, the effects of parameter remain the same as before. Thus, in the long-run equilibrium, with workers' bargaining power no longer influenced by their debt ratio, the qualitative effects of parameter changes on the knowledge-to-physical capital ratio are no longer entirely the same as before; nonetheless, they remain unchanged for the debt ratio.

The simplification introduced by assuming $\lambda_3 = 0$ allows us to determine how parameter changes impact the long-run equilibrium rates of capacity utilization, output growth, and employment, without introducing significant ambiguities. These equilibrium values are determined by substituting (23)-(25) into (18) to derive capacity utilization, then substituting the resulting expression into (6) to obtain output growth, and finally inserting both the expression for the long-run equilibrium capacity utilization and (24) into (5) to calculate the employment rate.

In the long-run equilibrium, as shown in Appendix A.3.4, capacity utilization, u^{**}, exhibits a positive relationship with the labor share, which is now given by $\tilde{\sigma}^* = \lambda_1/\gamma(1-\lambda_2)$ (henceforth, we employ a tilde ($\sim$) to denote the long-run values of σ, h, and δ when $\lambda_3 = 0$). The long-run capacity utilization increases with the interest rate, the same result as in the short-run equilibrium. In effect, substituting $\tilde{\sigma}^*$ into (24) and (25) to obtain $\tilde{h}^*$ and $\tilde{\delta}^*$, respectively, we find that none of these three long-run equilibrium values depend on the interest rate. The value of u^{**} increases with workers' propensity to consume, ϕ, and hence decreases with the debt repayment coefficient, $1 - \phi$. Using (18) evaluated at $\lambda_3 = 0$, we find that an increase in ϕ exerts both a direct positive effect (the only effect that operates in the short-run equilibrium, where $\tilde{\sigma}$, $\tilde{h}$, and $\tilde{\delta}$ are given) and an indirect negative effect (via $\tilde{\delta}^*$) on u^{**}, with the direct effect

[8] As shown in Appendix A.2.1, the long-run equilibrium solution characterized by $\hat{h} = \hat{\delta} = \hat{\sigma} = 0$ is unique and locally stable provided the value of λ_3 in (19) is low enough to yield $\partial\hat{\sigma}/\partial\sigma < 0$. Therefore, $\lambda_3 = 0$ automatically satisfies this uniqueness and asymptotic stability condition.

outweighing the indirect. The response of u^{**} to a change in γ, the parameter capturing the strength of the accelerator effect in the rate of knowledge capital accumulation, is ambiguous. An increase in γ exerts a direct positive effect (the only effect that operates in the short-run equilibrium), an indirect positive effect (due to an increase in $\tilde{\delta}^*$), and an indirect negative effect (due to a decrease in $\tilde{\sigma}^*$) on u^{**}. Given that the relative strength of these opposing effects depend on the underlying parameter configuration, the net impact remains indeterminate. The response of u^{**} to a change in β, the parameter capturing the strength of the accelerator effect in the rate of physical capital accumulation, is also ambiguous. An increase in β produces a direct positive effect (the only effect observed in short-run equilibrium) and two indirect negative effects (via $\tilde{h}^*$ and $\tilde{\delta}^*$, as both fall) on u^{**}. Since the relative strength of these opposing effects depend on the underlying parameter configuration, the net impact remains indeterminate. The value of u^{**} decreases with the propensity to save of capitalists, s. In the short-run equilibrium, an increase in s produces a direct negative effect on capacity utilization by reducing aggregate demand formation. In the long-run equilibrium, even though such an increase in s produces an indirect positive effect by raising $\tilde{\delta}^*$, the net effect on u^{**} remains negative. Finally, recall that the short-run equilibrium capacity utilization in (18) does not depend on the parameters λ_1 and λ_2 capturing workers' bargaining power in the real wage dynamics in (19). This is because $\tilde{\sigma}$ (and hence the real wage), $\tilde{h}$, and $\tilde{\delta}$ are given in the short run. However, an increase in any of these parameters generates an indirect positive effect on both $\tilde{\sigma}^*$ and $\tilde{h}^*$, and an indirect negative effect on $\tilde{\delta}^*$; the net result is an increase in u^{**}.

Given that the long-run equilibrium features stationary values of $\tilde{\sigma}$, $\tilde{h}$, and $\tilde{\delta}$—and thus a stationary rate of capacity utilization measured by the output-to-physical-capital ratio—the long-run equilibrium output growth, g^{**}, can be obtained by substituting u^{**} into (6). Therefore, the same qualitative responses of u^{**} to parameter changes obtained above carry over to g^{**}, except regarding changes in β, the parameter that captures the strength of the accelerator effect in the rate of physical capital accumulation. While the sign of the response of u^{**} to a change in β is ambiguous, the response of g^{**} is unambiguously positive, as demonstrated in Appendix A.3.5. Meanwhile, the long-run equilibrium employment rate, e^{**}, can be obtained by substituting u^{**} and $\tilde{h}^*$ into (5). As shown in Appendix A.3.6, e^{**} increases with the interest rate, the propensity to consume of workers, ϕ (and hence decreases with the debt repayment coefficient, $1-\phi$), the parameter that captures the strength of the accelerator effect in the rate of knowledge capital accumulation, γ, and the parameter that captures the strength of the accelerator effect in the rate of physical capital accumulation, β. Meanwhile, e^{**} decreases with the propensity to save of capitalists. All these qualitative results for the employment rate are the same as in the short-run equilibrium. Aside from changes in γ or β, they also match the qualitative responses of long-run equilibrium capacity utilization to the same parameter shifts. Moreover, they align with the long-run equilibrium results for the output growth rate, except in the case of changes in γ. Yet, while the long-run equilibrium values of the rates of capacity utilization and output growth are positively influenced by the parameters λ_1 and λ_2, which capture workers' bargaining power in the real wage dynamics, the sign of the response of e^{**} to those parameters is ambiguous. Because the long-run employment rate is given by $e^{**} = u^{**}/\tilde{h}^*$, and higher values of λ_1 or λ_2 raise both u^{**} and $\tilde{h}^*$, the net effect is ambiguous—since the relative strength of these effects depends on the underlying parameter configuration, the overall impact remains indeterminate. We recall that all the results presented and discussed above—derived under the assumption that real wage growth is no longer negatively influenced by workers' debt ratio—are summarized in Table 3.

6 Conclusions

This paper examines the bidirectional relationship between debt-financed investment in knowledge capital formation by working households and the functional distribution of income between wages and profits. The dynamic analysis is carried out using a three-dimensional nonlinear system that features the labor share of income, the ratio of knowledge to physical capital (reflecting the composition of the economy's total capital stock or wealth), and the ratio of working households' debt to knowledge capital (interpreted as their leverage ratio) as state variables.

In the short-run equilibrium—determined for given or predetermined values of the state variables in the dynamical system—any change in parameters or predetermined variables that raises aggregate demand, such as an increase in the labor share of income, leads to a higher rate of capacity utilization. Similar qualitative results apply to the employment rate (which equals the utilization rate of knowledge capital), except in the case of changes in the knowledge-to-physical capital ratio.

In the long run, the growth rate of the labor share is positively influenced by the growth rate of the real wage and negatively by the growth rate of labor productivity, which is equivalent to the growth rate of knowledge capital. Real wage growth is, in turn, positively affected by the employment rate and labor productivity growth, and negatively by the ratio of workers' debt to knowledge capital. The long-run equilibrium is unique, with its stability ensured as long as the sensitivity of real wage growth to the workers' debt ratio is not excessively strong. In this equilibrium configuration, parameter changes that boost aggregate demand or strengthen workers' bargaining power in wage negotiations (with the exception of the parameter mediating the positive effect of the expected real wage income on investment in knowledge capital) result in a higher labor share and a lower workers' debt ratio. Therefore, while greater debt-financed investment in knowledge capital formation by working households raises the level of economic activity in the short-run equilibrium, it may not have the same stimulating effect—and may also fail to increase the labor share of income—in the long-run equilibrium.

Although the reliance on debt by working households to finance knowledge capital formation is not inherently unsustainable, its medium- to long-term viability depends on macroeconomic policies that bolster aggregate demand and preserve the labor share of income. In the absence of such supportive policies, rising household indebtedness may reduce the level of economic activity—reflected in lower rates of capacity utilization, labor employment, and output growth—and contribute to macroeconomic instability. As with physical capital and the labor force, the availability of knowledge capital does not automatically generate its own demand. The relevant policy implication is clear: expansionary aggregate demand policies are a necessary complement to expansionary aggregate supply measures based on knowledge capital formation.

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A Appendix

A.1 Short-run comparative statics

$$\frac{\partial u^*}{\partial \sigma} = (s+\phi+\gamma-1)\frac{(1-s)i\delta h}{\Omega^2} = (s+\phi+\gamma-1)\frac{u^*}{\Omega} > 0, \tag{26}$$

whose sign stems from the differential marginal propensities: $\phi > (1-s) \Rightarrow (s+\phi+\gamma-1) > 0$.

$$\frac{\partial u^*}{\partial \delta} = \frac{u^*}{\delta} > 0, \tag{27}$$

$$\frac{\partial u^*}{\partial h} = \frac{u^*}{h} > 0, \tag{28}$$

$$\frac{\partial u^*}{\partial i} = \frac{u^*}{i} > 0, \tag{29}$$

$$\frac{\partial u^*}{\partial s} = \frac{-i\delta h\Omega - (1-\sigma)(1-s)i\delta h}{\Omega^2} < 0, \tag{30}$$

$$\frac{\partial u^*}{\partial \beta} = \frac{u^*}{\Omega} > 0, \tag{31}$$

$$\frac{\partial u^*}{\partial \gamma} = \frac{\sigma u^*}{\Omega} > 0, \tag{32}$$

$$\frac{\partial u^*}{\partial \phi} = \frac{\sigma u^*}{\Omega} > 0. \tag{33}$$

	Effect on u^*	**Effect on e^***
h	$+$	0
δ	$+$	$+$
σ	$+$	$+$
β	$+$	$+$
γ	$+$	$+$
ϕ	$+$	$+$
i	$+$	$+$
s	$-$	$-$

Table 1: Summary of short-run comparative statics ("0" indicates "no effect")

A.2 Long-run dynamics

A.2.1 Stability analysis

Given that u^*/h is not a function of h, and u^*/δ is not a function of δ, these are the elements of the Jacobian matrix of partial derivatives evaluated at the unique stationary configuration:

$$J_{11} = \frac{\partial \hat{h}}{\partial h} = -\beta\frac{\partial u^*}{\partial h} = -\beta\frac{u^*}{h^*} < 0, \tag{34}$$

$$J_{12} = \frac{\partial \hat{h}}{\partial \delta} = \left(\frac{\gamma\sigma^*}{h^*} - \beta\right)\frac{\partial u^*}{\partial \delta} = 0, \tag{35}$$

$$J_{13} = \frac{\partial \hat{h}}{\partial \sigma} = \left(\frac{\gamma\sigma^*}{h^*} - \beta\right)\frac{\partial u^*}{\partial \sigma} + \gamma\frac{u^*}{h^*} = \gamma\frac{u^*}{h^*} > 0, \tag{36}$$

$$J_{21} = \frac{\partial \hat{\delta}}{\partial h} = 0, \tag{37}$$

$$J_{22} = \frac{\partial \hat{\delta}}{\partial \delta} = -\frac{\gamma \sigma^*}{h^*}\frac{\partial u^*}{\partial \delta} = -\frac{\gamma \sigma^*}{h^*}\frac{u^*}{\delta^*} < 0, \tag{38}$$

$$J_{23} = \frac{\partial \hat{\delta}}{\partial \sigma} = -\frac{i}{\sigma^*} - \frac{i}{u^*}\frac{\partial u^*}{\partial \sigma} < 0, \tag{39}$$

$$J_{31} = \frac{\partial \hat{\sigma}}{\partial h} = 0, \tag{40}$$

$$J_{32} = \frac{\partial \hat{\sigma}}{\partial \delta} = \frac{1}{\delta^*}\left[\lambda_1 \frac{u^*}{h^*} + \lambda_2 \frac{\gamma \sigma^* u^*}{h^*} - \lambda_3 \delta^* - \frac{\gamma \sigma^* u^*}{h^*}\right] = 0, \tag{41}$$

$$J_{33} = \frac{\partial \hat{\sigma}}{\partial \sigma} = -\frac{\delta^*}{\sigma^* \Omega^*}[\lambda_1(1-s)i - \lambda_3(s-\beta)], \tag{42}$$

where $\Omega^* > 0$ denotes Ω evaluated at σ^*. The parameter configuration, including $s > \beta$, and the relevant values for the labor share, $0 < \sigma < 1$, together ensure that $\Omega \equiv s - \beta - (s + \phi + \gamma - 1)\sigma > 0$, which guarantees that the short-run capacity utilization in (18) is strictly positive and stable. The term in brackets in (42) is equal to the numerator of the long-run equilibrium labor share in (23), both of which are guaranteed to be positive given the parameter configuration. It follows that $J_{33} < 0$, which, as will be shown below, ensures that the unique long-run equilibrium given by (23)-(25) is locally stable. Thus, higher values of λ_1, i, and β—and lower values of s and λ_3—contribute to stability of the long-run equilibrium. In particular, $\lambda_3 = 0$ is sufficient (though not necessary) to guarantee stability. In matrix form:

$$J_{h^*,\delta^*,\sigma^*} = \begin{bmatrix} - & 0 & + \\ 0 & - & - \\ 0 & 0 & - \end{bmatrix} \tag{43}$$

The Routh-Hurwitz stability conditions for the three-dimensional dynamical system formed by (20)-(22) are as follows:

1. $Tr(J) < 0$,
2. $Det(J) < 0$,
3. $Det(J_1) + Det(J_2) + Det(J_3) > 0$, and
4. $Det(J) - Tr(J)[Det(J_1) + Det(J_2) + Det(J_3)] > 0$.

Solving:

1. $Tr(J) = J_{11} + J_{22} + J_{33} < 0$.

2. $Det(J) = J_{11}J_{22}J_{33} < 0$.

3. $Det(J_1) = J_{11}J_{22} > 0$, $Det(J_2) = J_{11}J_{33} > 0$, and $Det(J_3) = J_{22}J_{33} > 0$. It then follows that $Det(J_1) + Det(J_2) + Det(J_3) > 0$.

4.
$$Det(J) - Tr(J)[Det(J_1) + Det(J_2) + Det(J_3)] =$$
$$= -J_{1,1}[Det(J_1) + Det(J_2)] - (J_{2,2} + J_{3,3})[Det(J_1) + Det(J_2) + Det(J_3)] > 0.$$

A.2.2 Comparative statics for σ^*

$$\frac{\partial\sigma^*}{\partial\lambda_1} = \frac{(1-s)i}{(1-s)i\gamma(1-\lambda_2) - \lambda_3(s+\phi+\gamma-1)} = \frac{(1-s)i}{\Sigma} > 0, \tag{44}$$

where $\Sigma = (1-s)i\gamma(1-\lambda_2) - \lambda_3(s+\phi+\gamma-1) > 0$ is the denominator of σ^* in (23).

$$\frac{\partial\sigma^*}{\partial\lambda_2} = \frac{[\lambda_1(1-s)i - \lambda_3(s-\beta)](1-s)i\gamma}{\Sigma^2} = \frac{\sigma^*(1-s)i\gamma}{\Sigma} > 0, \tag{45}$$

$$\frac{\partial\sigma^*}{\partial\lambda_3} = -\frac{\Omega^*}{\Sigma} < 0, \tag{46}$$

$$\frac{\partial\sigma^*}{\partial\beta} = \sigma^*_\beta = \frac{\lambda_3}{\Sigma} > 0, \tag{47}$$

$$\frac{\partial\sigma^*}{\partial\phi} = \sigma^*_\phi = \frac{[\lambda_1(1-s)i - \lambda_3(s-\beta)]\lambda_3}{\Sigma^2} = \frac{\sigma^*\lambda_3}{\Sigma} > 0, \tag{48}$$

$$\frac{\partial\sigma^*}{\partial\gamma} = \sigma^*_\gamma = \frac{-[(1-s)i(1-\lambda_2) - \lambda_3]\sigma^*}{\Sigma} < 0, \tag{49}$$

whose sign can be determined by rewriting $\Sigma = \gamma[(1-s)i(1-\lambda_2) - \lambda_3] - \lambda_3[\phi - (1-s)] > 0$. As $\phi > (1-s)$, the term $[(1-s)i(1-\lambda_2) - \lambda_3]$ is positive, hence the negative sign of (49).

$$\frac{\partial\sigma^*}{\partial i} = \sigma^*_i = \frac{[\lambda_1(1-s)] - \sigma^*[(1-s)\gamma(1-\lambda_2)]}{\Sigma} = \frac{\lambda_3\Omega^*}{i\Sigma} > 0, \tag{50}$$

$$\frac{\partial\sigma^*}{\partial s} = \sigma^*_s = -\lambda_3\frac{\Omega^* + (1-\sigma^*)(1-s)}{\Sigma(1-s)} < 0. \tag{51}$$

A.2.3 Comparative statics for h^*

$$\frac{\partial h^*}{\partial\beta} = -\frac{\gamma}{\beta^2}\Big[\frac{\lambda_1(1-s)i - \lambda_3 s}{\Sigma}\Big], \tag{52}$$

$$\frac{\partial h^*}{\partial\gamma} = -\frac{\sigma^*}{\beta\Sigma}\lambda_3[\phi - (1-s)] < 0. \tag{53}$$

A.2.4 Comparative statics for δ^*

$$\frac{\partial \delta^*}{\partial \sigma^*} = -\frac{s(\phi+\gamma)}{\gamma\sigma^*(1-s)} - \frac{\delta^*}{\sigma^*} < 0, \tag{54}$$

$$\frac{\partial \delta^*}{\partial \phi} = -\frac{s\sigma^*}{\gamma\sigma^*(1-s)} - \frac{s(\phi+\gamma)}{\gamma\sigma^*(1-s)}\sigma^*_\phi - \frac{\delta^*}{\sigma^*}\sigma^*_\phi < 0, \tag{55}$$

$$\frac{\partial \delta^*}{\partial \beta} = -\frac{1}{\gamma\sigma^*(1-s)} - \frac{s(\phi+\gamma)}{\gamma\sigma^*(1-s)}\sigma^*_\beta - \frac{\delta^*}{\sigma^*}\sigma^*_\beta < 0, \tag{56}$$

$$\frac{\partial \delta^*}{\partial s} = \frac{[1-(\phi+\gamma)\sigma^*]}{\gamma\sigma^*(1-s)} - \frac{s(\phi+\gamma)}{\gamma\sigma^*(1-s)}\sigma^*_s - \frac{\delta^*}{\sigma^*}\sigma^*_s + \frac{\delta^*}{(1-s)} > 0, \tag{57}$$

$$\frac{\partial \delta^*}{\partial \gamma} = -\frac{s}{\gamma\sigma^*(1-s)}[\sigma^* + (\phi+\gamma)\sigma^*_\gamma] - \frac{\delta^*}{\sigma^*\gamma}(\gamma\sigma^*_\gamma + \sigma^*) > 0. \tag{58}$$

The sign in (58) is obtained recalling that $\sigma^*_\gamma < 0$, which implies, as shown below, that $(\gamma\sigma^*_\gamma + \sigma^*) < 0$, which implies in turn that $[\sigma^* + (\phi+\gamma)\sigma^*_\gamma] < 0$:

$$\gamma\sigma^*_\gamma + \sigma^* =$$

$$= \frac{\sigma^*}{\Sigma}\{-\gamma[(1-s)i(1-\lambda_2)-\lambda_3]+\gamma[(1-s)i(1-\lambda_2)-\lambda_3]-\lambda_3[\phi-(1-s)]\} = -\frac{\lambda_3[\phi-(1-s)]\sigma^*}{\Sigma} < 0.$$

	Effect on σ^*	**Effect on h^***	**Effect on δ^***
λ_1	$+$	$+$	$-$
λ_2	$+$	$+$	$-$
λ_3	$-$	$-$	$+$
β	$+$	?	$-$
ϕ	$+$	$+$	$-$
γ	$-$	$-$	$+$
i	$+$	$+$	0
s	$-$	$-$	$+$

Table 2: Summary of long-run comparative statics with $\lambda_3 > 0$ ("0" indicates "no effect"; "?" indicates "indeterminate effect")

A.3 Long-run comparative statics with $\lambda_3 = 0$

A.3.1 Comparative statics for $\tilde{\sigma}^*$

$$\tilde{\sigma}^* = \frac{\lambda_1(1-s)i}{(1-s)i\gamma(1-\lambda_2)} = \frac{\lambda_1}{\gamma(1-\lambda_2)}, \tag{59}$$

$$\frac{\partial \tilde{\sigma}^*}{\partial \lambda_1} = \frac{1}{\gamma(1-\lambda_2)} = \frac{\tilde{\sigma}^*}{\lambda_1} > 0, \tag{60}$$

$$\frac{\partial \tilde{\sigma}^*}{\partial \lambda_2} = \frac{\lambda_1}{\gamma(1-\lambda_2)^2} = \frac{\tilde{\sigma}^*}{(1-\lambda_2)} > 0, \tag{61}$$

$$\frac{\partial \tilde{\sigma}^*}{\partial \gamma} = \tilde{\sigma}^*_\gamma = -\frac{\lambda_1}{\gamma^2(1-\lambda_2)} = -\frac{\tilde{\sigma}^*}{\gamma} < 0. \tag{62}$$

A.3.2 Comparative statics for $\tilde{h}^*$

$$\frac{\partial \tilde{h}^*}{\partial \beta} = -\frac{\gamma\tilde{\sigma}^*}{\beta^2} < 0, \tag{63}$$

$$\frac{\partial \tilde{h}^*}{\partial \gamma} = \frac{\partial}{\partial \gamma}\left\{\frac{\gamma}{\beta}\frac{\lambda_1}{\gamma(1-\lambda_2)}\right\} = 0. \tag{64}$$

A.3.3 Comparative statics for $\tilde{\delta}^*$

$$\frac{\partial \tilde{\delta}^*}{\partial \tilde{\sigma}^*} = \tilde{\delta}^*_{\tilde{\sigma}^*} = -\frac{s(\phi+\gamma)}{\gamma\tilde{\sigma}^*(1-s)} - \frac{\tilde{\delta}^*}{\tilde{\sigma}^*} < 0, \tag{65}$$

$$\frac{\partial \tilde{\delta}^*}{\partial \phi} = -\frac{s}{\gamma(1-s)} < 0, \tag{66}$$

$$\frac{\partial \tilde{\delta}^*}{\partial \beta} = -\frac{1}{\gamma\tilde{\sigma}^*(1-s)} < 0, \tag{67}$$

$$\frac{\partial \tilde{\delta}^*}{\partial s} = \frac{[1-(\phi+\gamma)\tilde{\sigma}^*]}{\gamma\tilde{\sigma}^*(1-s)} + \frac{\tilde{\delta}^*}{(1-s)} > 0, \tag{68}$$

$$\frac{\partial \tilde{\delta}^*}{\partial \gamma} = -\frac{s}{\gamma\tilde{\sigma}^*(1-s)}[\tilde{\sigma}^* + (\phi+\gamma)\tilde{\sigma}^*_\gamma] - \frac{\tilde{\delta}^*}{\gamma\tilde{\sigma}^*}(\gamma\tilde{\sigma}^*_\gamma + \tilde{\sigma}^*) = -\frac{s\phi\tilde{\sigma}^*_\gamma}{\gamma\tilde{\sigma}^*(1-s)} > 0. \tag{69}$$

The sign in (69) is obtained by easily noting that $\gamma\tilde{\sigma}^*_\gamma + \tilde{\sigma}^* = 0$.

A.3.4 Comparative statics for u^{**}

$$u^{**} = \frac{is[1-(\phi+\gamma)\tilde{\sigma}^*] - \beta}{\beta\tilde{\Omega}^*}. \tag{70}$$

$$\frac{\partial u^{**}}{\partial \tilde{\sigma}^*} = \frac{i(s-\beta)(1-s)}{\beta(\tilde{\Omega}^*)^2}(\phi+\gamma-1) > 0, \tag{71}$$

where $\tilde{\Omega}^* > 0$ denotes Ω evaluated at $\tilde{\sigma}^*$.

$$\frac{\partial u^{**}}{\partial i} = u_i^{**} = \frac{(1-s)(\tilde{\delta}^*\tilde{h}^*)}{\tilde{\Omega}^*} > 0, \tag{72}$$

$$\frac{\partial u^{**}}{\partial \phi} = u_\phi^{**} = \frac{\tilde{\sigma}^*}{\beta\tilde{\Omega}^*}\Big[(1-s)i + (\phi+\gamma-1)\tilde{\sigma}^*\frac{u^{**}}{\tilde{h}^*\tilde{\delta}^*}\Big] > 0, \tag{73}$$

$$\frac{\partial u^{**}}{\partial \gamma} = -\frac{\tilde{\sigma}^*(1-s)i}{\beta\gamma(\tilde{\Omega}^*)^2}\Big[s(\phi+\gamma-1) - s\gamma(1-\tilde{\sigma}^*) + \beta(1-\phi)\Big], \tag{74}$$

$$\frac{\partial u^{**}}{\partial \beta} = \frac{(1-s)i}{\tilde{\Omega}^*}\Big[\frac{(\tilde{\delta}^*\tilde{h}^*)}{\tilde{\Omega}^*} - \frac{s[1-(\phi+\gamma)\tilde{\sigma}^*]}{\beta^2(1-s)}\Big], \tag{75}$$

$$\frac{\partial u^{**}}{\partial s} = u_s^{**} = -\frac{i\tilde{\sigma}^*(\phi+\gamma-1)}{\beta(\tilde{\Omega}^*)^2}\Big\{[1-(\phi+\gamma)\tilde{\sigma}^*] - \beta\Big\} < 0. \tag{76}$$

A.3.5 Comparative statics for g^{**}

$$g^{**} = \beta u^{**} = \frac{is[1-(\phi+\gamma)\tilde{\sigma}^*] - \beta}{\tilde{\Omega}^*}. \tag{77}$$

$$\frac{\partial g^*}{\partial \beta} = \frac{i\tilde{\sigma}^*}{(\tilde{\Omega}^*)^2}(\phi+\gamma-1)(1-s) > 0. \tag{78}$$

A.3.6 Comparative statics for e^{**}

$$e^{**} = \frac{u^{**}}{\tilde{h}^*} = \frac{(1-s)i\tilde{\delta}^*}{s(1-\tilde{\sigma}^*) + (1-\phi-\gamma)\tilde{\sigma}^* - \beta} = \frac{(1-s)i\tilde{\delta}^*}{\tilde{\Omega}^*}. \tag{79}$$

$$\frac{\partial e^{**}}{\partial \tilde{\sigma}^*} = \frac{(1-s)i\tilde{\delta}^*_{\tilde{\sigma}^*}}{\tilde{\Omega}^*} + (s+\phi+\gamma-1)\frac{u^{**}}{\tilde{\Omega}^*\tilde{h}^*}, \tag{80}$$

$$\frac{\partial e^{**}}{\partial i} = \frac{u_i^{**}}{\tilde{h}^*} - \frac{u^{**}}{(\tilde{h}^*)^2}\frac{\partial \tilde{h}^*}{\partial i} = \frac{u_i^{**}}{\tilde{h}^*} > 0, \tag{81}$$

$$\frac{\partial e^{**}}{\partial \gamma} = \frac{(1-s)i}{[\gamma(\tilde{\Omega}^*)]^2}\Big\{(1-\phi)[(s-\beta) - (\phi+\gamma)\tilde{\sigma}^*] + s\phi\tilde{\sigma}^*\Big\} > 0, \tag{82}$$

$$\frac{\partial e^{**}}{\partial \phi} = \frac{u^{**}_{\phi}}{\tilde{h}^*} - \frac{u^{**}}{(\tilde{h}^*)^2}\frac{\partial \tilde{h}^*}{\partial \phi} = \frac{u^{**}_{\phi}}{\tilde{h}^*} > 0, \tag{83}$$

$$\frac{\partial e^{**}}{\partial \beta} = \frac{i}{\gamma\tilde{\sigma}^*(\tilde{\Omega}^*)^2}(1-s)(\phi+\gamma-1)\sigma^* = \frac{i}{\gamma(\tilde{\Omega}^*)^2}(1-s)(\phi+\gamma-1) > 0, \tag{84}$$

$$\frac{\partial e^{**}}{\partial s} = \frac{u^{**}_{s}}{\tilde{h}^*} - \frac{u^{**}}{(\tilde{h}^*)^2}\frac{\partial \tilde{h}^*}{\partial s} = \frac{u^{**}_{s}}{\tilde{h}^*} < 0. \tag{85}$$

The signs in (81), (83), and (85) are obtained by noting that $\partial\tilde{h}^*/\partial i = \partial\tilde{h}^*/\partial\phi = \partial\tilde{h}^*/\partial s = 0$.

	Effect on $\tilde{\sigma}^*$	**Effect on** $\tilde{h}^*$	**Effect on** $\tilde{\delta}^*$	**Effect on** u^{**}	**Effect on** g^{**}	**Effect on** e^{**}
λ_1	+	+	−	+	+	?
λ_2	+	+	−	+	+	?
β	0	−	−	?	+	+
ϕ	0	0	−	+	+	+
γ	−	0	+	?	?	+
i	0	0	0	+	+	+
s	0	0	+	−	−	−

Table 3: Summary of long-run comparative statics with $\lambda_3 = 0$ (“0” indicates “no effect”; “?” indicates “indeterminate effect”)